Name: ___________________________________ Date: _________________

A2CC Finding Equations of Exponential Functions

Warm Up:

1. If $f^{-1}(x) = 4x + 3$ then which of the following is the correct formula for $f(x)$?

(a) $f(x) = \frac{1}{4}x - 3$

(b) $f(x) = \frac{1}{4}x - \frac{3}{4}$

(c) $f(x) = -4x - 3$

(d) $f(x) = 4x - 3$

2. The function $f(x)$ is an even function with $f(3) = 7$ and $f(9) = 11$. What is the average rate of change of $f(x)$ over the interval $-3 \le x \le 9$?

One of the skills that you acquired in Common Core Algebra I was the ability to write equations of exponential functions if you had information about the starting value and base (multiplier or growth constant). Let's review a very basic problem.

***Exercise* #1:** An exponential function of the form $f(x) = a(b)^x$ is presented in the table below. Determine the values of *a* and *b* and explain your reasoning.

x	0	1	2	3
$f(x)$	5	15	45	135

$a =$ _________

$b =$ _________

Final Equation: ____________________ Explanation:

Finding an exponential equation becomes much more challenging if we do not have output values for inputs that are increasing by unit values (increasing by 1 unit at a time). Let's start with a basic problem.

***Exercise* #2:** For an exponential function of the form $f(x)=a(b)^x$, it is known that $f(0)=8$ and $f(3)=1000$.

(a) Use the fact that $f(0)=8$ to determine the value of *a*. Show your thinking.

(b) Use you answer from part (a) and the fact that $f(3)=1000$ to set up an equation to solve for *b*. You will solve for *b* in part (c).

(c) Solve for the value of *b* using properties of exponents.

(d) Determine the value of $f(2)$

***Exercise* #3:** An exponential function exists such that $f(4)=3$ and $f(6)=48$, which of the following must be the value of its base? Explain or illustrate your thinking.

(1) $b=16$

(3) $b=6$

(2) $b=2$

(4) $b=4$

Now, let's work with the most generic type of problem. Just like with lines, **any two (non-vertically aligned) points will uniquely determine the equation of an exponential function**.

***Exercise* #4:** An exponential function of the form $y = a(b)^x$ passes through the points $(2, 36)$ and $(5, 121.5)$.

(a) By substituting these two points into the general form of the exponential, create a **system of equations** in the constants *a* and *b*.

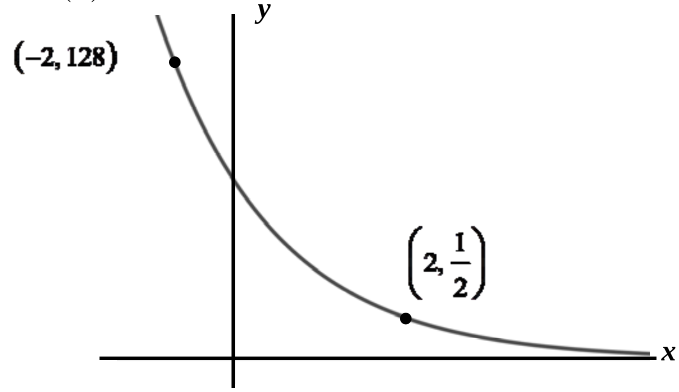
(b) Divide these two equations to eliminate the constant *a*. Recall that when dividing to like bases, you subtract their exponents.

(c) Solve the resulting equation from (b) for the base, *b*.

(d) Use your value from (c) to determine the value of *a*. State the final equation.

Let's now get some practice on this with a decreasing exponential function.

***Exercise* #5:** Find the equation of the exponential function shown graphed below. Be careful in terms of your exponent manipulation. State your final answer in the form $y = a(b)^x$.



***Exercise* #6:** A bacterial colony is growing at an exponential rate. It is known that after 4 hours, its population is at 98 bacteria and after 9 hours it is 189 bacteria. Determine an equation in $y = a\left(b\right)^x$ form that models the population, *y*, as a function of the number of hours, *x*. At what percent rate is the population growing per hour?

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HOMEWORK

1. For each of the following coordinate pairs, find the equation of the exponential function, in the form $y = a(b)^x$ that passes through the pair. Show the work that you use to arrive at your answer.

   (a) $(0, 10)$ and $(3, 80)$ (b) $(0, 180)$ and $(2, 80)$

2. For each of the following coordinate pairs, find the equation of the exponential function, in the form $y = a(b)^x$ that passes through the pair. Show the work that you use to arrive at your answer.

   (a) $(2, 192)$ and $(5, 12288)$ (b) $(1, 192)$ and $(5, 60.75)$

3. Each of the previous problems had values of *a* and *b* that were rational numbers. They do not need not be. Find the equation for an exponential function that passes through the points $(2, 14)$ and $(7, 205)$ in $y = a(b)^x$ form. When you find the value of *b* do not round your answer before you find *a*. Then, find both to the nearest *hundredth* and give the final equation. Check to see if the points fall on the curve.

4. A population of koi goldfish in a pond was measured over time. In the year 2002, the population was recorded as 380 and in 2006 it was 517. Given that y is the population of fish and x is the number of years *since* 2000, do the following:

(a) Represent the information in this problem as two coordinate points.

(b) Determine a linear function in the form $y = mx + b$ that passes through these two points. Don't round the linear parameters (m and b).

(c) Determine an exponential function of the form $y = a(b)^x$ that passes through these two points. Round b to the nearest hundredth and a to the nearest *tenth*.

(d) Which model predicts a larger population of fish in the year 2000? Justify your work.

5. Engineers are draining a water reservoir until its depth is only 10 feet. The depth decreases exponentially as shown in the graph below. The engineers measure the depth after 1 hour to be 64 feet and after 4 hours to be 28 feet. Develop an exponential equation in $y = a(b)^x$ to predict the depth as a function of hours draining. Round a to the nearest integer and b to the nearest hundredth. Then, graph the horizontal line $y = 10$ and find its intersection to determine the time, to the nearest tenth of an hour, when the reservoir will reach a depth of 10 feet.